$$N_\phi = \{ (M_1 + N_1 t^2 / 12a)/a \} \cos\phi \tag{23a}$$

$$M_\phi = M_o + (M_1 + N_1 t^2 / 12a) \cos\phi \tag{23b}$$

III. Discussion

From the harmonic analysis of the previous section it can be seen that for any value of n there is a nonzero value for the equilibrium strain which, following superposition, will lead to quite a complex function for the general case. For the stress, however, it has been shown that the equilibrium state for a harmonic greater than or equal to two is zero. Hence for any initial stress N_ϕ', M_ϕ' the equilibrium stress is as given by Eq. (23) where M_0, N_1, and M_1 are, respectively, the mean moment and first harmonics of the initial stress N_ϕ', M_ϕ'.

References

[1] Timoshenko, S. P., *Theory of Plates and Shells*, McGraw-Hill, New York, 1959.

[2] Dym, C. L., *Introduction to the Theory of Shells*, Pergamon Press, New York, 1974.

Timoshenko's Conjecture on Buckling of Annular Plates under Uniform External Pressure

G. K. Ramaiah* and K. Vijayakumar†

Indian Institute of Science, Bangalore, India

I. Introduction

ABOUT four decades ago Timoshenko,[1] conjectured while discussing Meissner's[2] results for buckling of annular plates under external pressure that for large holes the plate with clamped outer and free inner edges buckles in many circumferential waves. His comment was based on the physical argument that the conditions in such a plate (ring) are analogous to those of a long compressed rectangular plate clamped along one side and free along the other.

The previous conjecture was verified much later through numerical results by Rozsa[3] and recently by other investigators.[4-7] In Refs. 3 and 4, certain correlations which are different, one from the other, have been proposed between the physical parameters of the annulus and those of the infinite strip.

The object of the present Note is to offer a mathematical explanation for Timoshenko's conjecture and extend it to various combinations of clamped, simply supported, and free edge conditions. In addition, new correlations through a limiting process are derived. These are found to be useful in obtaining, a priori, a starting value for the number of circumferential waves in the critical buckling mode of a given annulus.

II. Analysis

A thin annular plate of uniform thickness h under external pressure p_0 is considered (see Fig. 1a), and the material of the plate is assumed to be homogeneous and isotropic. The prebuckling membrane stresses in such a plate are given by the well-known Lame's solutions. For a plate buckling in n circumferential waves, the lateral deflection $W(r,\theta)$ is represented in the from

$$W = W_n(r) \cos(n\theta + \epsilon) \tag{1}$$

Then the strain energy V of bending, and the work done T by the mid-plane forces during bending assume the forms

$$V = \frac{\pi}{2} D \int_a^b \Big[\Big(\frac{d^2 W_n}{dr^2} + \frac{1}{r}\frac{dW_n}{dr} - \frac{n^2}{r^2} W_n \Big)^2 - 2(1-\nu)\frac{d^2 W_n}{dr^2}\Big(\frac{1}{r}\frac{dW_n}{dr} - \frac{n^2}{r^2} W_n \Big) + 2(1-\nu)\frac{n^2}{r^2}\Big(\frac{dW_n}{dr} - \frac{W_n}{r} \Big)^2 \Big] r\,dr \tag{2}$$

and

$$T = \frac{\pi p_0 b^2 h}{2 b^2 - a^2} \Big[\int_a^b \Big(1 - \frac{a^2}{r^2}\Big)\Big(\frac{dW_n}{dr}\Big)^2 r\,dr + \int_a^b \Big(1 + \frac{a^2}{r^2}\Big)\Big(\frac{n}{r} W_n\Big)^2 r\,dr \Big] \tag{3}$$

in which ν is the Poisson's ratio, and $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate. The eigenmodes and eigenloads are governed by the variational problem $\delta(V-T)=0$ with respect to arbitrary variation δW_n satisfying relevant geometric boundary conditions.

The above-mentioned variational problem is expressed in more convenient form by using the transformations[6]

$$W_n(r) = r^2 w(r) \quad \text{and} \quad y = (r-a)/(b-a) \tag{4}$$

in succession. The integrand in the problem thus modified is found to be a fifth-order polynominal in the parameter $(b-a)/a$ with an eigenvalue parameter properly defined. In the limit $a \mapsto b$, the problem reduces to

$$\delta \Big[\int_0^1 \Big(\frac{1}{\beta}\frac{d^2 w}{dy^2} - \beta w \Big)^2 dy + 2(1-\nu) \int_0^1 \Big\{ W \frac{d^2 w}{dy^2} + \Big(\frac{dw}{dy}\Big)^2 \Big\} dy - \lambda \int_0^1 w^2 dy \Big] = 0 \tag{5}$$

in which

$$\beta = n(b-a)/a$$

and

$$\lambda = 2(p_0 b^2 h / D)(b-a)/(b+a)$$

The above variational problem Eq. (5) is identical to that of the buckling of a long rectangular strip of length a' and width b' if one takes $\lambda = \sigma h b'^2 / D$ and $\beta = m\pi b'/a'$ where σ is compressive stress at the shorter edges, and m is number of half-waves along the length of the strip (see Fig. 1b). By taking $b' = (b-a)$, as in Refs. 3 and 4, the correlation between σ and p_0 becomes

$$\sigma = 2 p_0 b^2 / (b^2 - a^2) \tag{6}$$

which is equal to the compressive hoop stress at the inner edge of the annulus. It may be mentioned here that σ in the corresponding correlation in Ref. 3 is equal to the average hoop stress along radial edge, and in Ref. 4, it is equal to the hoop stress at the outer edge.

The least eigenvalue λ for various values of β are obtained by the Rayleigh-Ritz method with simple polynomials in y as

Received Dec. 16, 1975; revision received April 29, 1976.

Index category: Structural Stability Analysis.

*CSIR Postdoctoral Research Fellow, Department of Aeronautical Engineering.

†Assistant Professor, Department of Aeronautical Engineering.

Table 1 Values of λ_0 and β_0 for variousboundary conditions, $\nu = 0.3$.

	Clamped-clamped	Clamped-free	Clamped-simply supported	Simply supported simply supported
λ_0	68.8	12.64	53.4	$4\pi^2$
β_0	4.75	1.92	3.90	π

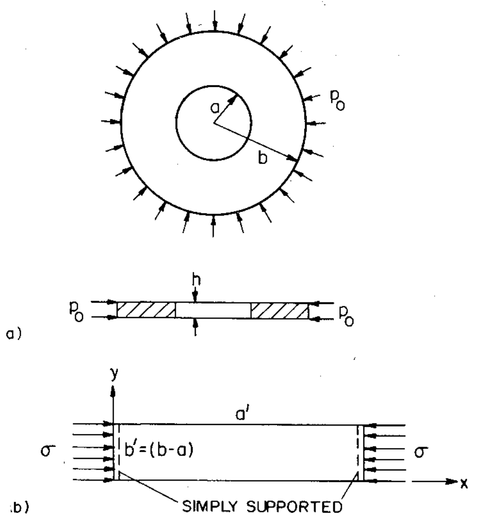

Fig. 1 a) Annular plate under uniform external pressure; b) rectangular strip under uniform compression.

$$X = x/a', \quad Y = y/b', \quad W(X,Y) = w(Y) \sin m\pi X$$

$$(V-T) = \frac{D}{2}\frac{1}{b'^4}\left[\int_0^1 \left\{\left(\frac{d^2 w}{dY^2} - \left(\frac{m\pi b'}{a'}\right)^2 w\right)^2 + 2(1-\nu)\left(\frac{m\pi b'}{a'}\right)^2 \times \left(w\frac{d^2 w}{dY^2} + \left(\frac{dw}{dY}\right)^2\right) - \frac{\sigma h b'^2}{D}\left(\frac{m\pi b'}{a'}\right)^2 w^2\right\} dY\right]$$

admissible functions. The curve λ vs β is then plotted from which the value of β at which λ assumes its minimum is obtained. The values of λ and β thus obtained are denoted by λ_0 and β_0 and are given in Table 1.

Note that in deriving the limiting variational problem Eq. (5), the first and the higher-order terms of $(b-a)/a$ in the modified energy expressions are neglected in the case of annulus, whereas no such approximation is involved in the strip problem. Hence the limiting solutions $\sigma h b'^2/D = \lambda_0$ and $m\pi b'/a = \beta_0$ in the strip problem can be interpreted in a useful manner for some finite strips. Whereas no such interpretation is possible in the case of annular plates, since the limiting solutions $(2p_0 b^2 h/D)(b-a)/(b+a) = \lambda_0$ and $n(b-a)/a = \beta_0$ are exact only in the limit $a \to b$. Nevertheless, the latter solution is useful in the sense that the value $\beta_0 a/(b-a)$ gives an indication of the number of circumferential waves (n) to be expected in the critical buckling mode of an annular plate.

References

[1] Timoshenko, S. P., *Theory of Elastic Stability*, McGraw-Hill, New York, 1936.

[2] Meissner, E., "Ueber das Knicken kreisringformiger Scheiben," *Schweizerische Bauzeitung*, Vol. 101, No. 8, 1933, pp. 87-89.

[3] Rozsa, M., "Stability Analysis of Thin Annular Plates Compressed Along the Outer or Inner Edge by Uniformly Distributed Radial Forces," *Acta Technica Academiae Scientiarum Hungaricae*, Vol. 53, 1966, pp. 359-377.

[4] Majumdar, S., "Buckling of a Thin Annular Plate under Uniform Compression," *AIAA Journal*, Vol. 9, Sept. 1971, pp. 1701-1707.

[5] Phillips, J. S. and Carney III, J. F., "Stability of an Annular Plate Reinforced with a Surrounding Edge Beam," *Journal of Applied Mechanics, Transactions of the ASME*, Vol. 41, No. 2, 1974, pp. 497-501.

[6] Ramaiah, G. K. and Vijayakumar, K., "Elastic Stability of Annular Plates under Uniform Compressive Forces along the Outer Edge," *AIAA Journal*, Vol. 13, June 1975, pp. 832-834.

[7] Ramaiah, G. K., "Some Investigations on Vibration and Buckling of Polar Orthotropic Annular Plates," Ph.D. thesis, May 1975, Indian Institute of Science, Bangalore, India.

Shrouds for Aerogenerators

O. Igra*
Ben-Gurion University of the Negev, Beer Sheva, Israel

Introduction

WIND is a natural energy resource that has heretofore not been widely used because of low energy content. Since the density of atmospheric air is approximately only one-thousandth that of water, a stream of air has only one-thousandth the energy contained in a stream of water of the same flow crosssection and velocity. Furthermore, it can be shown that at maximum efficiency an ideal windmill can use only 59.3% of the energy available in the stream tube that covers the windmill blades while the rest is carried downstream with the wind.[1,2] In real facilities, the efficiency is of course even less, due to aerodynamic and mechanical losses. Past attempts to exploit wind power for generating electricity have used windmills with giant blades in order to obtain practicable amounts of energy.[2-4] For example, in the Smith-Putnam project, the blade diameter was 175 ft.[3] In such an installation, the rotor must turn very slowly, resulting in design problems for the gears necessary to connect the windmill with the generator. Furthermore, in order to ensure a relatively stable power output under "off-design" conditions, it is necessary to use a rotor equipped with a blade pitch control and to keep its axis always parallel to the free wind direction. As a result, in the past, wind power was not economically competitive with other energy sources. To use wind power efficiently, reduce the size of rotors, and to increase the rotational speeds, various combinations of turbines operating inside specially designed shrouds have been investigated in Israel.[5-7] The investigated shrouds (one is shown schematically in Fig. 1) were composed of a bell-shaped intake, a cylindrical section, and a diffuser.

The purpose of the present paper is to present a new approach to shroud design, such that good aerodynamic performance is retained, while the shroud is made more attractive economically. Figure 1 summarizes the results obtained from the first model of the shroud.[6,7] The power ratio r is defined as

$$r = \frac{\frac{1}{2}\rho q_t^{\,3} A_t C_D}{0.593\,\frac{1}{2}\rho q_\infty^{\,3} A_t} = \frac{27}{16}\left(\frac{q_t}{q_\infty}\right)^3 C_D$$

C_D, the turbine load factor is defined as

$$C_D = \frac{P_1 - P_2}{\frac{1}{2}\rho q_t^{\,2}}$$

Received Jan. 22, 1976; presented as Paper 76-181 at the AIAA 14th Aerospace Sciences Meeting, Washington, D. C., Jan. 26-28, 1976; revision received March 24, 1976. This research was supported by the U.S.-Israel Binational Science Foundation.

Index category: Nozzle and Channel Flow

*Department of Mechanical Engineering.